CARNOT CYCLE ON DIFFERENT SCALES

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ABSTRACT

The present review highlights the detailed description of Carnot cycle on pressure-volume, pressure-entropy, temperature-pressure, volume-temperature, entropy-volume, internal energy-temperature and temperature entropy scale. The Carnot cycle involve four different steps a) isothermal reversible expansion, b) adiabatic reversible expansion, c) isothermal reversible compression and d) adiabatic reversible compression. Carnot cycle and its working in different articles and books were mainly given on pressure-volume scale only. On each of seven scales (P-V, P-S, T-P, V-T, S-V, U-T & T-S) the overall reversible Carnot cycle is explained by combining four different reversible steps.
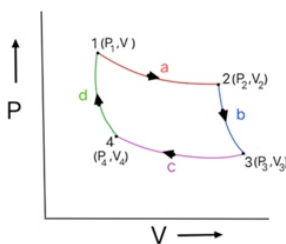
KEYWORDS: Carnot Cycle, Adiabatic, Isothermal, Expansion, Compression, Entropy.

1. INTRODUCTION

Carnot cycle illustrates the whole series of processes that occurs in an ideal (Carnot) heat engine in the form of a closed cycle called the Carnot cycle, developed in the 19th century by the French engineer Sadi Carnot[1]. The Carnot cycle has the maximum possible efficiency for any heat engine working between a high and a low temperature[2], so it is used as a standard for measuring the efficiencies of the other heat engines based on the same principle[1,3]. The most basic way in which the Carnot cycle is illustrated is by using the P-V graph. Apart from this, it can also be expressed via plotting the different state functions: P, V, T, U, H and S, against each other[4,5]. To the best of our knowledge there are no simple satisfactory explanations present that explains the shape and graph of these cycles[6]. So, we attempt to explain the different Carnot cycles using a simple mathematical and graphical approach which will help the researchers, teachers and students to understand the concept in a better way.

2. Different scales like pressure-volume, pressure-entropy, temperature-pressure, volume-temperature, entropy-volume, internal energy-temperature and temperature entropy have been used to describe the working of Carnot cycle with four steps a) isothermal reversible expansion, b) adiabatic reversible expansion, c) isothermal reversible compression and d) adiabatic reversible compression respectively.

2.1 P-V graph for Carnot cycle:

Graph 1: P-V graph for Carnot cycle

The Carnot cycle on P-V scale is given in Fig. 1[4].

Where, I: Isothermal Reversible Expansion

II: Adiabatic Reversible Expansion

III: Isothermal Reversible Compression

IV: Adiabatic Reversible Compression

2.1.1 Explanation:

I) For process a (Isothermal Reversible Expansion): It is known that for an isothermal reversible expansion process P(Pressure) and V(volume) are related as:

$$PV = \text{constant} \qquad \text{(By Boyle's Law)}$$

Therefore, $P \propto \frac{1}{V}$

So when volume increases, in expansion pressure decreases[4,5].

Thus the graph of this process is shown in Graph 1 (a)[7].

II) For process b (Adiabatic Reversible Expansion): It is known that for this process, P and V are related as[4,5]:

$$PV^\gamma = \text{constant}$$

Therefore, $P \propto \frac{1}{V^\gamma}$ and since γ is >1, P decreases more rapidly as compared to process a, which as shown in the Graph 1 (b)[4,5,7].

III) For process c (Isothermal Reversible Compression): It is known that for this process[4]:

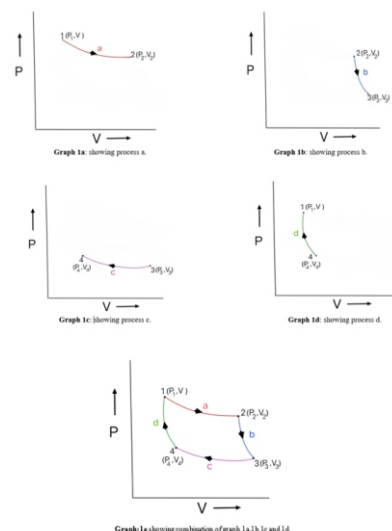$$PV = \text{constant by Boyle' law}$$

Therefore, $P \propto \frac{1}{V}$

Therefore, as volume decreases (due to compression), pressure increases which is as shown in Graph 1 (c)[4,7].

IV) For process d (Adiabatic Reversible Compression): It is known that for this process[4],

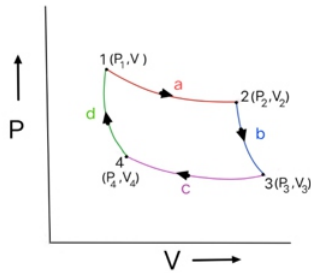$$PV^\gamma = \text{constant}$$

Therefore, $P \propto \frac{1}{V^\gamma}$

But in this process pressure increases much more rapidly as compared to the earlier process c because volume decreases due to compression as the value of γ is greater than 1. The graph for this process can be shown in Graph 1 (d)[7]. Therefore, on combining all the plots on *P-V* scale the overall reversible Carnot cycle is represented in graph 1 (e).

Graph 1a: showing process a.

Graph 1b: showing process b.

Graph 1c: showing process c.

Graph 1d: showing process d.

Graph 1e showing combination of graph 1a,1b,1c and 1d.

2.2 P-S graph for Carnot cycle[4]

Graph 1e: showing combination of graph 1a, 1b, 1c and 1d

Where, **a**: Isothermal Reversible Expansion

      b: Adiabatic Reversible Expansion

      c: Isothermal Reversible Compression

      d: Adiabatic Reversible Compression

2.2.1 Explanation:

I) For process a (Isothermal Reversible Expansion): It is known that for an ideal gas[4,5]: -

$$q = nRT\, ln\left(\frac{V_2}{V_1}\right) = nRT ln\left(\frac{P_1}{P_2}\right)$$

$$\Delta S = \frac{q}{T} = nR ln\left(\frac{P_1}{P_2}\right)$$

Since, $P_2 < P_1$ (because expansion is taking place in this process)

Therefore $\frac{P_1}{P_2} > 1$

$$ln\left(\frac{P_1}{P_2}\right) = +ve$$

$\Delta S = +ve$, means entropy increases logarithmically with the decrease in pressure[7].

The Graph 1 (a) is shown for this process[7]. The shape of the graph can also be understood as:

$$\Delta S = nR ln\left(\frac{P_1}{P_2}\right)$$

$$\Delta S = -nR ln\left(\frac{P_2}{P_1}\right)$$

$$-\frac{\Delta S}{nR} = ln\left(\frac{P_2}{P_1}\right)$$

Therefore, $\frac{P_2}{P_1} = e^{-\frac{\Delta S}{nR}}$

$$P_2 = P_1 e^{-\frac{\Delta S}{nR}}$$

This equation is similar to what is obtained in Rutherford and Soddy Law in radioactive decay:

$N = N_0\, e^{-\lambda t}$ (where N is the number of atoms and t is the time) [8], in which the graph is similar to the case which is as shown in Graph 2 (i).

II) For the process b (Adiabatic Reversible Expansion): It is known that for an adiabatic process

$$q = 0$$

Therefore, $\Delta S = 0$, means there is no change in entropy [4].

Therefore, S = constant, when pressure decreases (due to expansion) as shown in graph 2 (b)[7].

III) For the process c (Isothermal Reversible Compression): For this process the change in entropy is [5]

$$\Delta S = nR ln\left(\frac{P_3}{P_4}\right) \text{ (as in process } a)$$
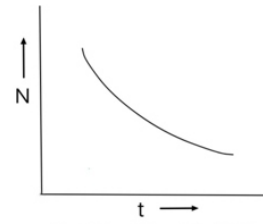
But since, there is compression i.e. $P_3 < P_4$

Therefore, $\frac{P_3}{P_4} < 1$, implies that $ln\left(\frac{P_3}{P_4}\right) = -ve$

Therefore, $\Delta S = -ve$, means entropy is decreasing logarithmically with the increase in pressure (compression) the graph 2 (c) is obtanied[7].
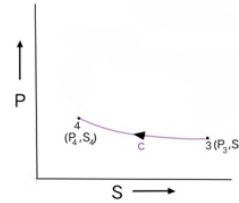
IV) For process d (Adiabatic Reversible Compression): $q = 0$

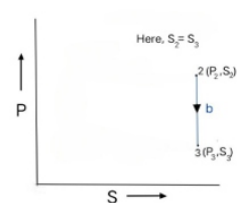Therefore, $\Delta S = 0$, implies that S = constant [4]

Pressure increases due to compression the graph 2 (d) is obtained[7]. Therefore, on combining all the plots on P-S scale the overall reversible Carnot cycle is represented in graph 2 (e).
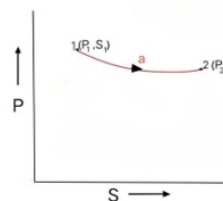

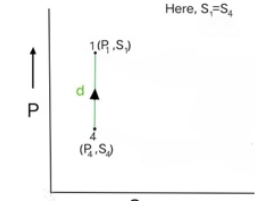Graph 2i: showing the variation of N Vs t
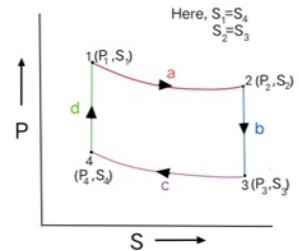

Graph 2a: showing process a.
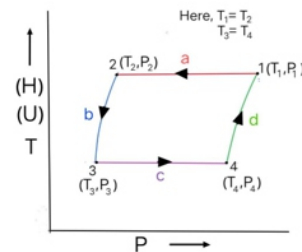

Graph 2b: showing process b.

Graph 2c: showing process c.

Graph 2d: showing process d.

Graph:2e showing combination of graph 2a, 2b, 2c and 2d.

1.3 T-P graph for Carnot cycle[4]:

Graph 3: T-P graph for Carnot cycle

Where, **a**: Isothermal Reversible Expansion

      b: Adiabatic Reversible Expansion

      c: Isothermal Reversible Compression

      d: Adiabatic Reversible Compression

2.3.1 Explanation:

I) For process a (Isothermal Reversible Expansion): for this process dT = 0 [4]

T = constant

Therefore, $\Delta H = 0$ and $\Delta U = 0$

This implies that H and U both are also constant when P decreases (expansion). The Graph 3 (a) obtained as shown.

II) For process b (Adiabatic Reversible Expansion): It is known that P and T are related as[4]:

$$P^{1-\gamma} \times T^{\gamma} = constant, \text{ where } \gamma = C_P/C_V$$

Therefore $P^{\frac{1-\gamma}{\gamma}} \times T = constant$

$$T = \frac{C}{P^{\frac{1-\gamma}{\gamma}}} = C \times P^{\frac{\gamma-1}{\gamma}}$$

$$0 < \frac{\gamma-1}{\gamma} < 1$$

Thus, in this process the temperature increases or decreases in a curved manner as the value of pressure increases or decreases (note in this case decreases as P decreases since there is expansion) as shown in graph 3 (b)[7]. The shape of the graph can be understood by using the following few hypothetical values.

For examples:

A) Let $\gamma = 1.5$ (average value for most gases under normal conditions)

Therefore $T = constant \times P^{1.5-1/1.5}$

1) Let $P = 6.00$ Pa

   T= $1.82 \times C$ K

2) Let P = 5.00 Pa

   T= $1.71 \times C$ K

3) Let P = 4.00 Pa

   T = $1.59 \times C$ K

4) Let P = 3.00 Pa

   T = $1.44 \times C$ K

5) Let P= 2.00 Pa

   T= $1.26 \times C$ K

6) Let P =1.00 Pa

   T = $1.00 \times C$ K

This hypothetical data is summarized in the table 3 (i) and the corresponding graph is given as 3 (i). Note that the shape of the graph at other values of γ (which are closer to 1.5 under normal conditions)[9] are similar as the powers of P will be closer

to 1/3 only, so they follow the similar trends by approximation[7]. As examples the hypothetical data when $\gamma = 1.4$ and $\gamma = 1.6$ is given as:

b) When $\gamma = 1.4$, $T = constant \times P^{0.4/1.4}$

c) When $\gamma = 1.6$, $T = constant \times P^{0.6/1.6}$, the values of T at different values of P are summarized in the following tables (3ii and 3iii) and the corresponding graphs are shown as 3 (ii) and 3 (iii).

III) For process c (Isothermal Reversible Compression): dT = 0

$$T = Constant$$

Therefore, $\Delta H = 0$ and $\Delta U = 0$

This implies that H and U both are also constant when P increases (compression). The Graph 3 (c) is shown.

IV) For process d (Adiabatic Reversible Compression): Similar to process b but here since P increases therefore the value of T also increases (refer to the tables and graphs of process b), the curve is shown in graph 3 (d).

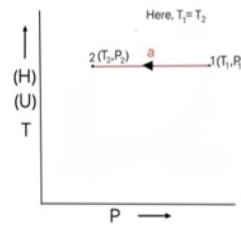*Note: In Adiabatic Reversible processes [i.e. process (b) and (d)]:

$$\Delta U = C_V \times \Delta T \text{ and } \Delta H = C_p \times \Delta T \text{ [4]}$$

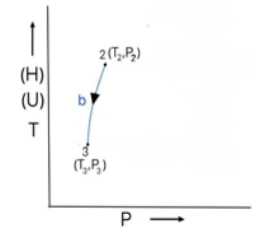Therefore, $\Delta U \propto \Delta T$ and $\Delta H \propto \Delta T$

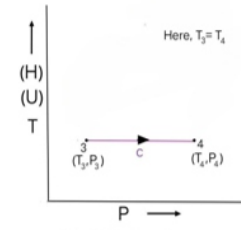   or $U \propto T$ and $H \propto T$

   or $U \propto H \propto T$

Thus similar trends are followed for U and H when they are plotted against P. Therefore, on combining all the plots on T-P scale the overall reversible Carnot cycle is represented in graph 3 (e).
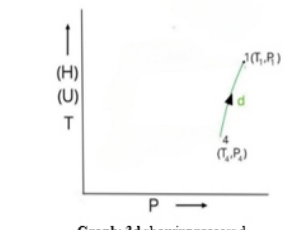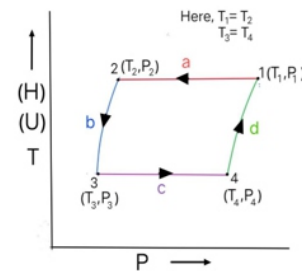

Graph: 3a showing process a

Graph: 3b showing process b

Graph: 3c showing process c

Graph: 3d showing process d

Graph: 3e showing combination of graph 3a, 3b, 3c and 3d

S.No.	P (in Pa)	T(in K) in terms of C
1	6.00	1.82
2	5.00	1.71
3	4.00	1.59
4	3.00	1.44
5	2.00	1.26
6	1.00	1.00

Table 3i: values of P and T when $T = C.P^{\frac{1}{3}}$

Graph 3i: showing T-P plot when $T = C.P^{\frac{1}{3}}$

S.No.	P (in Pa)	T(in K) in terms of C
1	6.00	1.67
2	5.00	1.58
3	4.00	1.49
4	3.00	1.37
5	2.00	1.22
6	1.00	1.00

Table 3ii: values of P and T when $T = C.P^{\frac{0.4}{1.4}}$

Graph 3ii: showing T-P plot when $T = C.P^{\frac{0.4}{1.4}}$

S.No.	P (in Pa)	T(in K) in terms of C
1	6.00	1.96
2	5.00	1.83
3	4.00	1.68
4	3.00	1.51
5	2.00	1.30
6	1.00	1.00

Table 3iii: values of P and T when $T = C.P^{\frac{0.6}{1.6}}$

Graph 3iii: showing T-P plot when $T = C.P^{\frac{0.6}{1.6}}$

2.4 V-T graph for Carnot cycle[4]:

Graph 4: V-T graph for Carnot cycle

Where, a: Isothermal Reversible Expansion

b: Adiabatic Reversible Expansion

c: Isothermal Reversible Compression

d: Adiabatic Reversible Compression

2.4.1 Explanation:

I) For process a (Isothermal Reversible Expansion): dT = 0

$$T = \text{constant}$$

Therefore, $\Delta H = 0$ and $\Delta U = 0$

This implies that H and U both are also constant when pressure decreases (expansion), is given in graph 4 (a)[4,7].

II) For process b (Adiabatic Reversible Expansion): It is known that

$$T \times V^{\gamma-1} = \text{constant} \,[4]$$

Therefore, $T = \dfrac{c}{V^{\gamma-1}}$

Since, $\gamma > 1$ for all gases, $\gamma - 1 > 0$ or +ve

Therefore, $T = \dfrac{c}{V^{+ve\,power}}$

In the expansion as volume increases, $\dfrac{c}{V^{\gamma-1}}$ decreases as and hence is the value of T decreases[7]. The shape of this graph given in Graph 4 (b) can be understood by using the following hypothetical examples:

A) Let $\gamma = 1.5$ (average value for most gases under normal conditions)

Therefore, $\gamma - 1 = 1.5 - 1 = 0.5$

1) Let V = 1.00 L

$T = \dfrac{c}{(1.00)^{0.5}} = 1.00 \times C\,K$

2) Let V = 2.00 L

$T = \dfrac{c}{(2.00)^{0.5}} = 0.71 \times C\,K$

3) Let V = 3.00 L

$T = \dfrac{c}{(3.00)^{0.5}} = 0.58 \times C\,K$

4) Let V = 4.00 L

$T = \dfrac{c}{(4.00)^{0.5}} = 0.50 \times C\,K$

5) Let V = 5.00 L
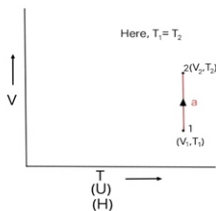
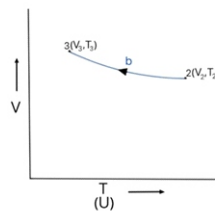$T = \dfrac{c}{(5.00)^{0.5}} = 0.45 \times C\,K$

5) Let V = 6.00 L

$T = \dfrac{c}{(6.00)^{0.5}} = 0.41 \times C\,K$

This data is as summarized in the Table 4 (i) and the corresponding plot in Graph 4 (i). Also since $U \propto H \propto T$ (refer to the note in process b in the P-T graph explanation) similar trends are followed for H and U. **Note:** That the shape of the graph at other values of γ are similar(which are closer to 1.5 under normal conditions [9]).as the powers of V will be closer to 0.5 only, so they follow the similar trends by approximation[7] . As examples the hypothetical data and the graphs when $\gamma = 1.4$ and $\gamma = 1.6$ are given below:
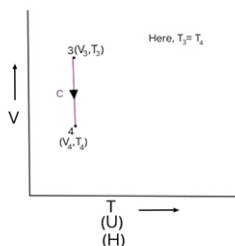
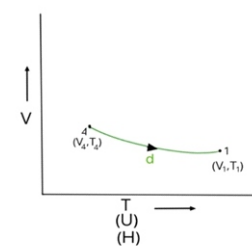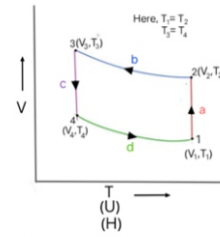b) When $\gamma = 1.4$, $T = \dfrac{c}{V^{0.4}}$

Graph 4a: showing process a

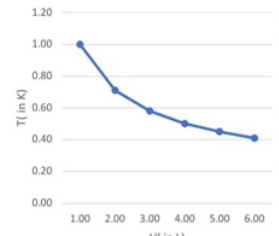Graph 4b : showing process b

Graph 4c: showing process c

Graph 4d: showing process d

Graph 4e: showing combination of graph 4a, 4b, 4c and 4d

S.No.	V (in L)	T(in K) in terms of C
1	1.00	1.00
2	2.00	0.71
3	3.00	0.58
4	4.00	0.50
5	5.00	0.45
6	6.00	0.41

Table 4i: values of V and T when $T = \dfrac{c}{V^{0.5}}$

Graph 4i: showing T-V plot when $T = \dfrac{c}{V^{0.5}}$

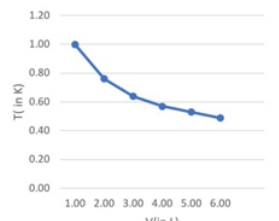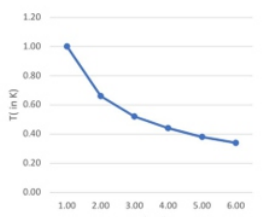| S.No.	V (in L)	T(in K) in terms of C
1	1.00	1.00
2	2.00	0.76
3	3.00	0.64
4	4.00	0.57
5	5.00	0.53
6	6.00	0.49

Table 4ii: values of V and T when $T = \dfrac{c}{V^{0.4}}$

Graph 4ii: showing T-V plot when $T = \dfrac{c}{V^{0.4}}$

S.No.	V (in L)	T(in K) in terms of C
1	1.00	1.00
2	2.00	0.66
3	3.00	0.52
5	4.00	0.44
5	5.00	0.38
6	6.00	0.34

Table 4iii: values of V and T when $T = \dfrac{c}{V^{0.6}}$

Graph 4iii: showing T-V plot when $T = \dfrac{c}{V^{0.6}}$

2.5 S-V graph for Carnot cycle[4] :

Graph 5: S-V graph for Carnot cycle

Where, a: Isothermal Reversible Expansion

b: Adiabatic Reversible Expansion

c: Isothermal Reversible Compression

d: Adiabatic Reversible Compression

2.5.1 Explanation:

I) For process a (Isothermal Reversible Expansion): It is known that for this process[5] :

$$q = nRT\ln\left(\frac{V_2}{V_1}\right)$$

$$\Delta S = \frac{q}{T} = nR\ln\left(\frac{V_2}{V_1}\right)$$

Since, expansion is there, $V_2 > V_1$

Therefore $\dfrac{V_2}{V_1} > 1$

This implies $\ln\left(\frac{V_2}{V_1}\right) = +ve$

$\Delta S = + ve$

Thus, S increases logarithmically[5,7]. The Graph 5 (a) is shown.

II) For process b (Adiabatic Reversible Expansion)[4]:

$q = 0$

$\Delta S = 0$ or $S =$ constant

V increases as there is expansion. The Graph 5 (b) is shown.

III) For process c (Isothermal Reversible Compression): Similar to process a[4]:

$$\Delta S = nRln\left(\frac{V_4}{V_3}\right)$$

Since there is compression, $V_4 < V_3$

Therefore , $\frac{V_4}{V_3} < 1$

This implies , $\ln\left(\frac{V_4}{V_3}\right) = -ve$

$\Delta S = -ve$

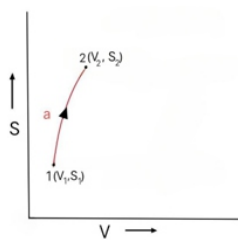entropy decreases logarithmically[7] as given in graph 5 (c).

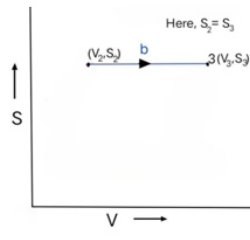IV) For the d (Adiabatic Reversible Compression): Similar to process b[4].
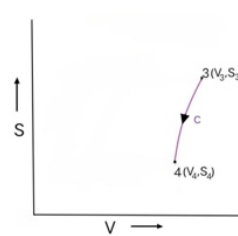
$q = 0$

$\Delta S = 0$ or $S =$ constant

volume decreases as there is compression. The Graph 4 (d) is shown. Therefore, on combining all the plots on S-V scale the overall reversible Carnot cycle is represented in graph 5 (e).

Where, **a**: Isothermal Reversible Expansion

b: Adiabatic Reversible Expansion

c: Isothermal Reversible Compression

d: Adiabatic Reversible Compression

2.6.1 Explanation:

I) For process a (Isothermal Reversible Expansion): It is known that for this process[4]:

$\Delta T = 0$ or $T =$ constant

$\Delta U = 0$ or $U =$ constant

$\Delta H = 0$ or $H =$ constant

This implies that a is a point. The Graph 6 (a) is as shown.

II) For process b (Adiabatic Reversible Expansion): It is known that

$U \propto H \propto T$ (refer to the note in process b of P-T graph)

As T decreases (during expansion), U and H decreases with respect to each other in a linear manner[7]. The Graph 6 (b) is shown.

III) For process c (Isothermal Reversible Compression): same as process a. As all T, U and H are constant the graph 6 (c) is shown as point.

IV) For process d (Adiabatic Reversible Compression): Similar to process b

$U \propto H \propto T$ (refer to the note in process b of P-T graph)

As T increases (during compression), U and H increases with respect to each other in a linear manner. The graph 6 (d) is shown. Therefore, on combining all the plots on U-T scale the overall reversible Carnot cycle is represented in graph 6 (e).
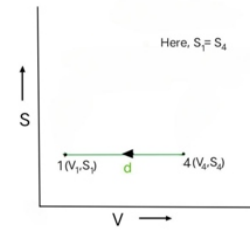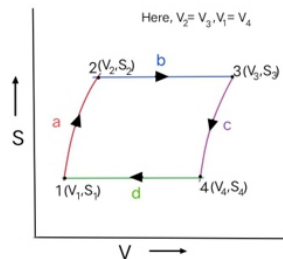

Graph 5a: showing process a.

Graph 5b: showing process b
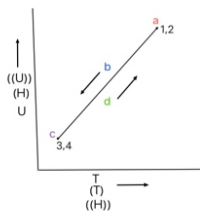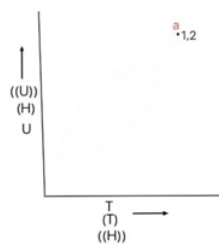

Graph 5c: showing process c.

Graph 5d: showing process d.

Graph 5e: showing combination of graph 5a, 5b, 5c and 5d

2. 6 U-T graph for Carnot cycle[4]:

Graph:6 U-T graph for Carnot cycle.

Graph 6a: showing process a.

Graph 6b: showing process b.

Graph 6c: showing process c.

Graph 6d: showing process d.

Graph 6e: showing combination of graph 6a, 6b, 6c and 6d

2.7 T-S graph for Carnot cycle[10]:

Graph 7: T-S graph for Carnot cycle.

Where, **a**: Isothermal Reversible Expansion

     b: Adiabatic Reversible Expansion

     c: Isothermal Reversible Compression

     d: Adiabatic Reversible Compression

2.7.1 Explanation:

I) For process a (Isothermal Reversible Expansion): It is known that for this process:

$$\Delta S = nRln\left(\frac{V_2}{V_1}\right)$$

Since, $V_2 > V_1$ (expansion), ΔS = +ve or entropy increases (refer to process a in S-V graph)[5].

But $\Delta T = 0$, $\Delta H = 0$ and $\Delta U = 0$ or T, H, S are constant. The Graph 7 (a) is shown.

II) For process b (Adiabatic Reversible Expansion): It is known that [4]

$$q = 0$$

Therefore, $\Delta S = 0$ or S = constant

T-V are related as : $T = \frac{c}{V^{\gamma-1}}$ (refer to V-T graph explanation)

So as V increases, T decreases.   (refer to V-T graph explanation)

Also since $U \propto H \propto T$ (refer to the P-T graph explanation), similar trends are followed for U and H when they are plotted against S. The plot is shown in graph 7 (b).

III) For process c (Isothermal Reversible Compression): Similar to process a

$$\Delta S = nRln\left(\frac{V_4}{V_3}\right)$$

Since, $V_4 < V_3$ (compression), ΔS = -ve or entropy decreases (refer to process c in S-V graph). But $\Delta T = 0$, $\Delta H = 0$ and $\Delta U = 0$ or T, H, S are constant as given in graph 7 (c).

IV) For process d (Adiabatic Reversible Compression): Similar to process c.

$$S = constant$$

$T = \frac{c}{V^{\gamma-1}}$, But here, since V decreases (compression), T increases.

Also since $U \propto H \propto T$, similar trends are followed for U and H when they are plotted against S.

The Graph 7 (d) is shown.Therefore, on combining all the plots on T-S scale the overall reversible Carnot cycle is represented in graph 7 (e).
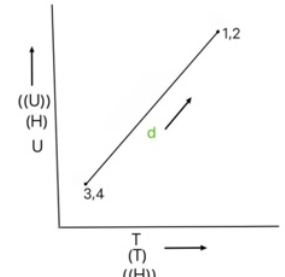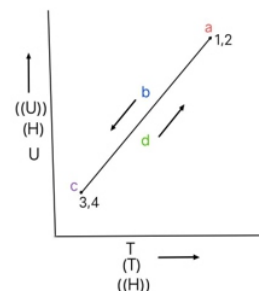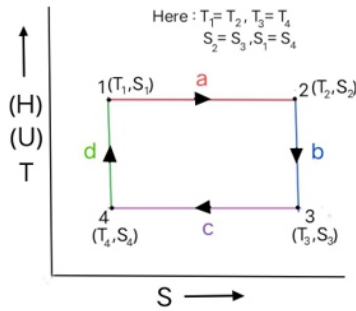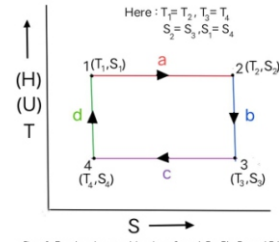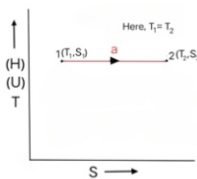

Graph 7a: showing process a.

Graph 7b: showing process b.

Graph 7c: showing process c.

Graph 7d: showing process d.

Graph 7e: showing combination of graph 7a, 7b, 7c and 7d

The total value of different functions involved in seven different scales are summarized here:

1) Total work: The work done in completing one Carnot cycle is given by [4]:

$$W_T = W_a + W_b + W_c + W_d$$

where

W_a = work done in process a (Isothermal Reversible Expansion)

W_b = work done in process b (Adiabatic Reversible Expansion)

W_c = work done in process c (Isothermal Reversible Compression)

W_d = work done in process d (Adiabatic Reversible Compression)

Therefore[4] ,

$$W_T = -nRT_1 ln\left(\frac{V_2}{V_1}\right) + \frac{1}{\gamma - 1}[P_3V_3 - P_2V_2] - nRT_3 ln\left(\frac{V_4}{V_3}\right) + \frac{1}{\gamma-1}[P_1V_1 - P_4V_4]$$

Using the relation, $TV^{\gamma-1}$ for the adiabatic processes b and d, it can be easily proven that: $\frac{V_4}{V_3} = \frac{V_1}{V_2}$. Also by applying Boyle's Law for the Isothermal processes a and c, we get: $P_1V_1 = P_2V_2$ and $P_3V_3 = P_4V_4$.

Therefore , $W_T = -nRT_1 ln\left(\frac{V_2}{V_1}\right) - nRT_3 ln\left(\frac{V_1}{V_2}\right) + \frac{1}{\gamma-1}[P_3V_3 - P_2V_2 + P_2V_2 - P_3V_3]$

$$W_T = -nRT_1 ln\left(\frac{V_2}{V_1}\right) + nRT_3 ln\left(\frac{V_2}{V_1}\right) + \frac{1}{\gamma-1}[P_3V_3 - P_2V_2 + P_2V_2 - P_3V_3]$$

$$W_T = nRln\left(\frac{V_2}{V_1}\right)[T_3 - T_1]$$

Also since $T_3 < T_1$ and $V_2 > V_1$ (refer the V-T graph for the Carnot cycle). $W = -ve$ Means on completing one Carnot cycle net work is done by the system

2) Total Heat: The total heat in completing one Carnot cycle is given by:

$$q_T = q_a + q_b + q_c + q_d$$

q_a = heat involved in process a (Isothermal Reversible Expansion)

q_b = heat involved in process b (Adiabatic Reversible Expansion)

q_c = heat involved in process c (Isothermal Reversible Compression)

q_d = heat involved in process d (Adiabatic Reversible Compression)

We know that b and d are adiabatic processes, therefore q_b and q_c both are = 0[4].

Therefore , $q_T = nRT_1 ln\left(\frac{V_2}{V_1}\right) + 0 + nRT_3 ln\left(\frac{V_4}{V_3}\right) + 0$

$q_T = nRT_1 ln\left(\frac{V_2}{V_1}\right) + nRT_3 ln\left(\frac{V_1}{V_2}\right)$,Since $\frac{V_4}{V_3} = \frac{V_1}{V_2}$

$q_T = nRT_1 ln\left(\frac{V_2}{V_1}\right) - nRT_3 ln\left(\frac{V_2}{V_1}\right)$

$q_T = nRln\left(\frac{V_2}{V_1}\right)[T_1 - T_3]$

Also since $T_3 < T_1$ and $V_2 > V_1$, q_T = +ve and $q_T = -W_T$

This means that total heat is absorbed by the system and the same is used for doing the work.

This also shows that Carnot Heat engine is an ideal heat engine in which total heat absorbed from the source = total work done by the engine.

3) Total Internal energy Change (ΔU_T): The total enthalpy change in one complete Carnot cycle is given by: $\Delta U_T = \Delta U_a + \Delta U_b + \Delta U_c + \Delta U_d$, where

ΔU_a = Internal energy change in process a (Isothermal Reversible Expansion)

ΔU_b = Internal energy change in process b (Adiabatic Reversible Expansion)

ΔU_c = Internal energy change in process c (Isothermal Reversible Compression)

ΔU_d = Internal energy change in process d (Adiabatic Reversible Compression)

We know that for Isothermal Reversible processes a and c, ΔU_a and ΔU_c both are = 0, [4]

Therefore. $\Delta U_T = 0 + C_V(T_3 - T_2) + 0 + C_V(T_1 - T_4)$

Now, $T_1 = T_2$ and $T_3 = T_4$ (because a and c are isothermal processes), we get

$\Delta U_T = C_V(T_4 - T_1) + (-C_V(T_4 - T_1))$

$\Delta U_T = 0$

4) Total enthalpy change (ΔH_T): Similar to ΔU_T but here instead of C_v we have C_p

$$\Delta H_T = \Delta H_a + \Delta H_b + \Delta H_c + \Delta H_d$$

$$\Delta H_T = 0 + C_p(T_3 - T_2) + 0 + C_p(T_1 - T_4)$$

$$\Delta H_T = C_p(T_4 - T_1) + (-C_p(T_4 - T_1)) = 0$$

5) Total entropy change (ΔS_T): The Total entropy change in one complete Carnot cycle is given by:

$$\Delta S_T = \Delta S_a + \Delta S_b + \Delta S_c + \Delta S_d, \text{ where}$$

ΔS_a = Entropy change in process a (Isothermal Reversible Expansion)

ΔS_b = Entropy change in process b (Adiabatic Reversible Expansion)

ΔS_c = Entropy change in process c (Isothermal Reversible Compression)

ΔS_d = Entropy change in process d (Adiabatic Reversible Compression)

We know that for the adiabatic processes b and d, ΔS_b and ΔS_d are = 0 (since q (heat) =0 in adiabatic processes)[5].

Therefore, $$\Delta S_T = nR\ln\left(\frac{V_2}{V_1}\right) + 0 + nR\ln\left(\frac{V_4}{V_3}\right) + 0$$

$$\Delta S_T = nR\ln\left(\frac{V_2}{V_1}\right) + nR\ln\left(\frac{V_1}{V_2}\right), \text{ Since } \frac{V_4}{V_3} = \frac{V_1}{V_2}$$

$$\Delta S_T = nR\ln\left(\frac{V_2}{V_1}\right) - nR\ln\left(\frac{V_2}{V_1}\right)$$

$$\Delta S_T = 0$$

6) Total Helmholtz free energy (ΔA_T): For one complete Carnot cycle, it is given by:

$$\Delta A_T = \Delta U_T - \Delta(TS_T) \quad [4]$$

$$\Delta A_T = \Delta U_T - (T\Delta S_T + S_T\Delta T)$$

Since for one complete Carnot cycle ΔU_T and ΔS_T are =0 (as proved above). Also $\Delta T=0$ because the initial and the final states are same so there is no net change in temperature of the system.

Therefore, $\Delta A_T = 0$

7) Total Gibbs free energy (ΔG_T): For one complete Carnot cycle, it is given by[4]:

$$\Delta G_T = \Delta H_T - \Delta(TS_T)$$

$$\Delta G_T = \Delta H_T - (T\Delta S_T + S_T\Delta T)$$

Since for one complete Carnot cycle ΔH_T and ΔS_T are =0 (as proved above). Also, $\Delta T=0$ because the initial and the final states are same so there is no net change in temperature of the system.

Therefore, $\Delta H_T = 0$

The values of different functions at different scales as explained is summarized in table A.

Table A. Values of different functions for one complete cycle

S. No	Function	Scale	Value for one complete cycle
1.	W_T	Pressure-volume	$W_T = nR\ln\left(\frac{V_2}{V_1}\right)[T_3 - T_1]$
2.	q_T	pressure-entropy	$q_T = nR\ln\left(\frac{V_2}{V_1}\right)[T_1 - T_3]$
3.	ΔU_T	temperature-pressure	Zero
4.	ΔH_T	volume-temperature	Zero
5.	ΔS_T	entropy-volume	Zero
6.	ΔA_T	internal energy-temperature	Zero
7.	ΔG_T	temperature entropy	Zero

CONCLUSION

The above results prove that the values of all state functions on temperature-pressure, volume-temperature, entropy-volume, internal energy-temperature and temperature entropy scales are zero for one complete Carnot cycle which is in accordance with the fact that when the initial and final states are same in a process. pressure-volume, pressure-entropy,

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